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IMU Fields Medal 2022
Hugo Duminil-Copin Interview

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When did you realize that you wanted to be mathematician? Who and what inspired you to become a mathematician? Was it an easy or difficult decision for you?

I do not quite remember when was the first time I wanted to be a mathematician. I have always been attracted to science, and fairly early got curious about what the work of a scientist was, but I was more interested in astronomy, and did not think of mathematicians as being a real job.

Towards the end of my high school and during my undergrad studies, it appeared clearly that I was very comfortable with math, but I was still hesitating between the interest that I could find in understanding physics phenomena, and the comfort I would get when facing the perfect fully rigorous solution of a mathematical puzzle.

When entering the École Normale Supérieure de Paris, I could choose to pursue both mathematics and physics but thought that this was a dangerous gamble for me to have more than one iron in the fire. I therefore chose to keep mathematics, with the original aim of becoming a math teacher.

It’s only after starting my Master degree that I felt in love with research. The field of statistical physics combined the rigor of probability theory, which was the object of my master, with the ability to satisfy my curiosity for physics.

From there, the path was pretty straightforward and I never regretted it. Overall, I cannot say that the decision to become a mathematician was difficult, as it matured sufficiently long to become an evidence at the time I made it.

What kind of mathematics did you like the best in school and at the university? Do you have a favorite math problem from back then?

I don’t think I had a favorite domain of mathematics when I was in school, but clearly had young loves during my undergrads. Students are introduced to a number of different fields of mathematics, and the basics in some of these fields, for instance group theory or linear algebra, are matured theories that reach a certain level of perfection. I guess almost every mathematician falls in love with one of these domains at some point. In my case, it was group theory. It lasted for a few months, but this was quickly overwhelmed by the diversity of the arguments that one finds in analysis.

Yet, the real shock for me was the discovery of probability theory. It exactly combined everything I loved in. It also lied at the crossroad of a number of other domains, which enabled me to make excursions to these neighboring fields once in a while.

How did you find "your" area of mathematics? What makes it particularly attractive for you?

In the École Normale Supérieure de Paris, I was initiated to probability theory by Jean-François Le Gall. His classes are known in France as flirting with perfection. The theory was developing before our eyes in an apparent flawless fashion. After seeing that, it became an evidence for me to specialize in probability for my last year of undergrads.

A second professor that deeply influenced my early years was Wendelin Werner. He gave us a class on statistical physics focusing on percolation and the Ising model. The class was axed on intuition and physical relevance. Somehow, this clicked perfectly with what I liked: a combination of mathematical rigor and physics intuition. Since this class, I am completely in love with this area of mathematics.

The beauty of statistical physics comes not only from its physical relevance, but also because it relies on very diverse mathematics, including probability but also algebra, geometry, analysis, combinatorics, etc. At the beginning of my PhD, I used discrete analysis on graphs, later on I was inspired by renormalization arguments in mathematical physics, and recently I got more and more interested in exactly integrable systems… But always by combining these ingredients with probability-type arguments. That’s what I liked the most about statistical physics.

How do you choose the problems to work on?

I am very sensitive on this point. It is practically impossible for me to work on a problem that it forced upon me. By the way, during my PhD I tried many alternative problems to the original subject of my advisor, and this is fortunate as I still don’t know how to solve this subject.

There are several things that are crucial for me when choosing a problem. Probably most importantly, I need to be esthetically pleased with both the result I try to prove and the type of mathematics that can be involved in its resolution. This is an absolute requirement for me to trigger creativity.

But often the beauty of the problem is not quite sufficient. I would say that I also like to follow long-term research programs that I decompose into a large number of incremental steps. I therefore like that the problems I work on is one of these incremental steps.

I think the combination of these two things – looking for esthetically appealing problems, but keeping in mind a long-term goal – works pretty well for me.

Let me confess a guilty pleasure: I like to revisit foundational results in areas of mathematics related to the research program I am working on. Of course, it usually leads nowhere, but once in a while one can find a new proof and as a by-product new tools to solve the problems we originally wanted to tackle.

Do you feel that the progress in mathematics is sometimes very fast and sometimes very slow? What do you when it goes fast? What do you when it is slow?

Doing mathematics requires to deal with times where research does not progress, and times where it rushes. Everybody has his own way to navigate between calm weather and storms.

My approach is to be fully available for research when storm is coming. In order to do that, I use the calm periods to advance as much as possible the routine work that we get to do as a mathematician: editorial work, refereeing, committees, writing papers, preparing classes, etc. In this spare time, I also try to widen the scope of my research by learning about new problems and techniques.

When the storm arrives, then the real challenge begins. Ideas usually come in groups, and rarely at the most practical moment. So, one needs to be organized and to proceed in steps. Of course, I cannot do that. I get overly excited and start heading in every direction at once, filling up pages and pages of drafts and I exchange dozens of e-mails with my collaborators. These are very intense and exhausting times but these are also the most beautiful ones. A scientist basically leaves for these moments, even though they require time to recover afterwards.

What was the first real mathematical result you obtained? Tell us about the result itself, the circumstances, and what effect it had on you.

One of the results I produced during my master thesis had an important impact on my vision of mathematics. I was doing an internship in UBC and was working on bootstrap percolation, a model of forest fires. I was trying to solve one of the main open problems in the area, which was to identify the value at which the three-dimensional model was undergoing a drastic change of behavior.

During the summer, I reached the conclusion that I had a full proof and started working on the paper. One day, while checking the arXiv (where mathematicians and physicists put their preprints in open access), I discovered a paper posted in the morning by Balogh, Bollobas, and Morris, solving exactly the problem I was dealing with. I was devastated.

Still, I thought that the two arguments had several differences, and that in particular the higher dimensional case was still falling within the framework of my proof. I therefore contacted the authors to tell them about this, but honestly had little to show but a loosely written draft that was nowhere close to be sufficiently polished. At this stage, the three authors reacted in a way that marked me for years. They also had an argument for the higher dimensional case, but proposed, rather than trying to race each other, to join forces, implementing an argument that was taking the best of the two approaches. This paper is now well recognized in the bootstrap percolation community, and was only the first paper of a fruitful and extremely enjoyable collaboration.

It was a perfect example of the merits of being generous and inclusive in our way of doing mathematics. These authors could have won the race without difficulty against a young inexperienced mathematician, but giving me a chance to be part of the team led to much more on both sides.

I always try to be inclusive and avoid competition at all cost, remembering how beneficial their gesture was for my career, and to share my ideas with whoever wishes. In the end, mathematics is much more fun when done together.

Can you tell us about your biggest "Eureka!" moment?

This is an easy question, as I vividly remember it. It does not concern my most important paper, but was clearly the most exciting moment of research I had the joy of experiencing.

It was a pretty standard day. We were chit chatting while taking coffee with Vincent Tassion, a friend and long-time collaborator. At some point, Vincent decided to tell me about a cool argument on Bernoulli percolation that he had with his former PhD advisor Vincent Beffara. What they were obtaining was interesting but it was mostly the argument that caught my attention.

Discussing with him and trying to understand it, we reached a reformulation that directly connected with an estimate that was relevant for another completely different problem I was working on at the time. We realized that this could give an answer to the problem.

More importantly, the way we were formulating this condition immediately triggered a reaction in Vincent's mind: the condition was trivial to obtain for Bernoulli percolation itself. With his observation, we immediately realized that we could provide a totally new and very short proof of one of the cornerstone results of Bernoulli percolation.

The whole discussion lasted ten minutes, and at the end of it, we had a full argument written on the blackboard. This proof is now taught in every course on percolation, as part of the basic curriculum of a master program in statistical physics.

What was remarkable in this story is that none of us was even trying to work on this problem. It is this aimless intellectual ping-pong that converged almost immediately into a complete novel and (in my opinion) quite powerful argument. Sharing this Eureka moment in a genuine way with somebody else is so much more exciting than any lonely discoveries I made.

I try as much as possible to reproduce this type of shared discoveries by discussing openly and often without premeditated goals with my collaborators. Several other events like that happened later on in my career, but this one was the first of the kind, and definitely the most enjoyable one.

What did you feel and what did you do when you learned about being awarded the prize?

Well, funnily enough, Carlos Kenig, head of IMU and of the Fields committee, tried to reach me by email to organize a phone call, but the email went directly to my spams. I was in addition discussing super intensively with Vincent Tassion on a problem dear to us for many years so I did not check my emails actively. As such it delayed the announcement. When I discovered the second email of Pr. Kenig asking for a meeting, I immediately understood that this was probably about the medal. A few hours later, I was on the phone with Pr. Kenig, who told me the wonderful news.

A feeling that occurred to me almost immediately was the great responsibility that the medal represents. I do not believe that it changes much the way we do our everyday work, but outside math it propels you among the ambassadors of your discipline. Being a good example for the rest of society is a challenge which is both difficult and a little bit scary, especially that the strongest mathematicians are not necessarily the best to talk about math outside our small circles. Yet, the award is such it is now my duty to do it, and I will therefore do my best not to disappoint the community.

Except for this instantaneous feeling of responsibility, I must have been slightly under shock as I remember that in the days that followed I was mostly thinking obsessively about the new arguments that we developed with Vincent during the previous days and not so much about the medal.

I was told about the award in the middle of January. The embargo was a challenge for me to fully enjoy the news. I usually like to share news with people I love but in this case, it is strictly forbidden. In addition to this, the situation between Russia and Ukraine deteriorated quickly around that time so I was clearly not in a state of mind compatible with enjoying the medal. But now that the news is out, I will at least be able to share the experience with my family, my collaborators, and my community, and to realize what really happened.

Who are the people who contributed the most to this success?

To me, research is a group experience. I like to work as a team and as such I share credit with my collaborators. Among my most regular collaborators, Ioan Manolescu and Vincent Tassion are by far the closest to me. A big chunk of the works I have done in the past fourteen years have been with one of them, and I am infinitely grateful for this fruitful collaboration. In fact, they are much more than collaborators: they are very close friends and I feel very fortunate to have them in my life. I definitely dedicate a big part of the medal to both of them.

Michael Aizenman also played an important role in my research. We represent two generations that work together, enabling us to benefit from our very different experiences. Research constantly evolves, and the way we do mathematical physics today differ from how it was done forty years ago. With Michael, we manage to interact very efficiently to obtain the best of our two visions of mathematics and physics. I also thank him tremendously.

Finally, I have a thought for Vladas Sidoravicius, who passed away tragically a few years ago. He was a dear friend and an important collaborator. Math is not as fun without him.

What are the new horizons, new problems, new goals for you now?

It is hard to know as lines are constantly moving. My co-authors and I definitely achieved a few steps towards the completion of several research programs that are dear to me. Since we are getting quite close to some of them, I definitely want to push even more towards them. And then, who knows where solving these problems will bring us.

During my career, I alternated times during which I was going deeper in certain directions, and times during which I tried to get wider. The past few years definitely corresponded to the first kind, and I am now feeling that after solving these programs, it will be time to pass to a period of exploration of new horizons. Off the top of my head, I am particularly interested in quantum chains and Anderson localization, to mention only two possible directions…

But today the priority is clearly to complete these programs that were initiated a long time ago.

Do you draw inspiration from teaching mathematics?

Teaching is one of the most important aspects of our job. I had the occasion not to teach in the Institut des Hautes Études Scientifiques, which is a pure research institution. Yet, I missed teaching way too much. The interactions with the students, the introspection necessary to prepare classes, the ideas that emerge from it, are very important to the mathematician. It is somehow the respiration of the mathematician: it allows her/ him to step back from research to oxygen her/his mind. In my case, I truly need this respiration to break long drought periods.

One should also not forget that teaching is the most direct impact mathematicians have on society, and as such is a very important component of our job.

Do you wish people knew more about what is happening in mathematics? Is there something mathematicians should do to help people appreciate the importance and beauty of what they do?

How to make people more interested in mathematics is a difficult question. What I noticed is that people judge how much they love math by evaluating their level when they were kids. This is rather strange, as there are many people that were bad in literature at school and still love to read. For math, this is different…

One of the main problems is that math is thought of as a purely utilitarian thing, which is at best useful for applications to our technology, and at worse restricted to the compound of school. There is nothing more wrong than this. One of the most important aspects of math is that it is one of our common and universal heritage as human, and that its beauty and enjoyment are not the preserve of a few knowledgeable persons.

Tremendous progress has been made in the popularization of math and a lot of it is aimed at showing the beauty of it rather than its usefulness. I think this is truly the right way to make people more interested. We will never reconcile our kids with math by telling them it is useful, but by making them enjoy it.

In fact, math has probably everything to become the perfect hobby: critical thinking, problem solving, interesting stories, links to other fields of science, historical relevance, etc. The important catch is to adapt the level to the audience.

How important are interactions with a computer for your work, now and in the future? Do you get more from interactions with a computer or interactions with people?

Of course computers are an important part of our job, but I guess you refer to recent advances in deep learning and in AI, and whether I can foresee some applications of computer support to my research.

While I have witnessed amazing recent applications of AI, for instance to representation theory, and that for many years simulations have been an important component of our job as mathematical physicists, in my case I do not really use computers and prefer to knock on my office neighbor’s door. That being said, the scientific interest of AI for unravelling new research directions and solving problems is not questionable anymore, and it is clear that it will play a more and more important role in our mathematical life.

Outside of mathematics, what are your favorite things to do, interests, pursuits?

I have a daily life that I would consider as fairly normal. I am doing the same as most people: watching series, taking care of the house, seeing friends. Of course, recently a new thing happened that changed my life completely: I became a father. Interacting with my daughter is by far my most favorite thing on Earth.

Do you approach them as a mathematician, or are you happy to forget about mathematics while you are on a break?

Being the son of a sports teacher and a former dancer, I liked to practice sports since childhood. I never pushed myself as much in sport as I do in math. For instance, I never tried to optimize or get better at a sport for the sake of it. This is a different story with math, for which I take time to think about the way I am doing it, what could be improved in my organization, my thinking process, in my interaction with my collaborators and the community, etc.

As a prize winner, you also become an ambassador of mathematics to the society and its leaders. What would say in a meeting with, for instance, a Science and Education Minister?

While I understand that a Fields medalist is necessarily invited to become an ambassador, I am not quite sure yet what I would tell a science and education minister. I do not think this prize makes my opinion on several subjects more valuable than before, and definitely not more accurate. I therefore want to check first with people involved in this for years what are the needs and requests that the majority would want to be conveyed, in order to be an efficient megaphone for the community.